Knitro for Nonlinear Optimal Power Flow Applications

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This paper considers the use of Knitro to solve optimal power flow (OPF) problems occurring in the electric power industry. Knitro is an embeddable software engine for solving large, nonlinear optimization problems. Its robustness, efficiency, and ease of use make it a powerful tool in Energy Management Systems (EMS), for both rapid operational computations and longer range planning. Knitro can optimize generator costs, line losses, and more, while satisfying nonlinear AC power flow equations and system constraints. Clients currently use Knitro on networks containing tens of thousands of buses.

Transmission networks are most accurately modeled with complex impedance parameters (e.g., IEEE Common Format), which lead to power flow equations of the form:

$$\begin{aligned} P_{G_k} - P_{L_k} &= \sum_{m=1}^{N} V_k V_m [G_{km} cos(\delta_k - \delta_m) + B_{km} sin(\delta_k - \delta_m)] \\ Q_{G_k} - Q_{L_k} &= \sum_{m=1}^{N} V_k V_m [G_{km} sin(\delta_k - \delta_m) - B_{km} cos(\delta_k - \delta_m)] \end{aligned}$$

where $Y_{km} = G_{km} + jB_{km}$ is the admittance between buses k and m, P_G and Q_G are active and reactive generated powers, and P_L and Q_L are active and reactive loads. Unknowns to be solved for are the voltages V_k, phase angles δ_k, and some or all of the power terms. Typical OPF problems also impose upper and lower bounds on voltages and powers, and may provide terms to model transformers, phase shifters, etc.

Knitro is designed for large-scale nonlinear problems, allowing EMS applications to solve the AC power flow equations instead of using linearized DC approximations. Knitro is unique among nonlinear solvers in providing both **Barrier** (interior point) methods and an **Active Set** method. OPF problems are best handled by a Barrier method that "crosses over" to the Active Set method, a procedure handled automatically by Knitro. Cross-over refines the interior Barrier method result into a highly accurate solution with precise sensitivities.

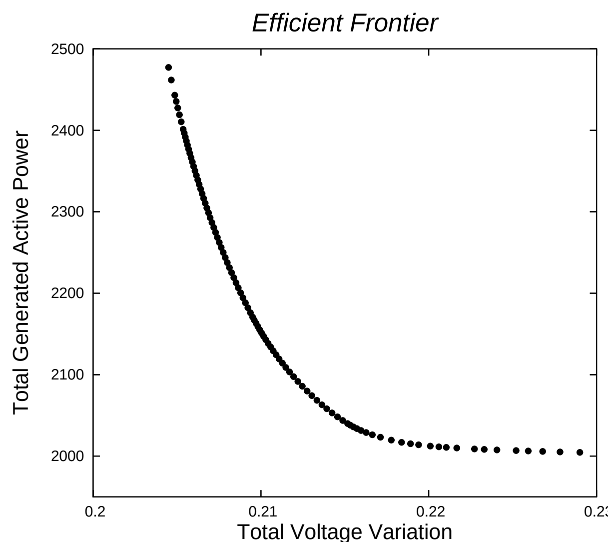
Knitro provides many tuning options to deal with a wide range of OPF applications. The solution method is a key option, and performance for particular problems can be tuned by trying two or three method-specific options (see the *Knitro User's Manual* for details). Computational costs typically increase with problem size, but Knitro minimizes the cost with sparse linear algebra techniques, CPU-specific BLAS implementations, and state-of-the-art algorithmic enhancements.

Optimal power flow problems, like most nonlinear problems, converge fastest if the application or modeling language computes second partial derivatives. If derivatives are not available, Knitro provides alternatives such as quasi-Newton approximations and built-in finite differencing.

To illustrate KNITRO's scalability, a two-stage stochastic programming problem was developed from an actual 1200 bus system. Quadratic generation costs are minimized over scenarios in which the load varies randomly. First stage decision variables are local power input levels, with power levels purchased from outside the system serving as second stage variables. Each scenario s has its own variables $V_{s,k}$ and $\delta_{s,k}$, so the deterministic equivalent problem size is roughly proportional to the number of scenarios. As the results below show, the Barrier Direct method takes a comparable number of iterations for any problem size. The cost of each iteration increases due to linear algebra overhead.

Scenarios	Variables	Iterations	CPU time
1	2,801	102	11 sec
10	25,292	62	133 sec
20	50,282	61	366 sec
30	75,272	62	349 sec
40	100,262	55	368 sec
50	125,252	148	1190 sec

As an example of KNITRO's flexibility, the efficient frontier of a multi-objective OPF problem was computed. Two conflicting goals are the minimization of active power generation and minimization of variation in line voltages V_k. To expore the tradeoffs, form a weighted combination of the two objectives and solve the optimization problem. The plot at right shows the best values that can be achieved as the weight parameter is varied. The KNITRO "restart" capability was used to solve these related problems 3 times faster than if they were each solved from scratch. Restart begins iterating from the previous solution, and reuses the problem structure since only the weight parameter is changed.



KNITRO is a premier solver for nonlinear optimization problems, handling bound constraints, nonlinear equalities and inequalities (both convex and nonconvex), and complementarity constraints. KNITRO solves large-scale NLPs, LPs, QPs, MPCCs, nonlinear systems of equations, and least squares problems. KNITRO is available as a thread-safe, embeddable software library on multiple platforms, with programmatic APIs and interfaces to major modeling languages. Full support and continued development of KNITRO is provided by Ziena Optimization, Inc. For more information, visit *http://www.ziena.com.*